Theoretical Evaluation of Angular Diversity in Multibeam Smart Antenna Systems

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***Abstract*— Smart antenna systems, especially those based on multibeam technology, could use angular diversity to mitigate the effects of fading on the received signal at the base station. This paper evaluates angular diversity, by expanding on a theoretical model to determine the correlation coefficient and power of signals from a four-beam antenna system. The performance of such a system with a four-branch and a two-branch maximal-ratio combining is explored under the condition of unequal branch signal strengths.**

I. Introduction

Commercial base stations use diversity reception to minimize the effects of Rayleigh fading on the received signal. Space and polarization diversity techniques are the most common forms of diversity reception. Another form of diversity, angular diversity, has not yet found wide deployment, but that could change with the advent of smart antenna systems, especially in their simplest form – the multibeam system.

A multibeam smart antenna system divides the coverage area of a typical base station sector into multiple narrow beams. The system selects the best beam to serve a particular mobile based on the strength and/or quality of the received signal (e.g. bit or symbol error rate). This results, on average, in a reduction of received interference at the base station proportional with the ratio of the half-power beamwidth of the narrow beam to that of the original sector antenna. A second-best beam may be selected for diversity reception. This describes angular diversity whereby the received signal is selected on the basis of angle of arrival.

To assess the performance of angular diversity, this paper analyzes the correlation and average power of signals in a representative four-beam antenna system. In Section II, a mathematical development establishes a process to quantify these measures for incident signals of varying angular spread. Section III presents an example of the correlation coefficient and average power of signals received by a four-beam antenna system. Section IV, discusses the performance of four-branch and two-branch maximal-ratio combining (MRC) under unequal branch signal-to-noise ratio (SNR) condition. Section V recaps with some practical aspects of deploying angular diversity techniques in a land mobile communication system.

II. Mathematical Development

We follow the process presented in [1] and [2] to derive equations for the correlation coefficient and power received by each beam of a multibeam antenna system. The normalized correlation coefficient for two complex signals E_1 and E_2 is given by

$$\rho = \frac{\left\langle E_1 E_2^* \right\rangle - \left\langle E_1 \right\rangle \left\langle E_2^* \right\rangle}{\sqrt{\left\langle E_1 E_1^* \right\rangle - \left\langle E_1 \right\rangle^2} \sqrt{\left\langle E_2 E_2^* \right\rangle - \left\langle E_2 \right\rangle^2}} . \qquad (1)$$

In the case of a multibeam antenna system, E_1 and E_2 are the signals induced at the ports of two beams. We state that these signals are the result of a superposition at the base station antenna of plane waves of vertically polarized electric fields

$$E_z = E_0 \sum_{k=1}^{K} C_k G(\phi_k) e^{j(\omega t + \alpha_k)} \qquad (2)$$

where ω is the carrier frequency, $E_0 C_k$ is the amplitude of E_z of the k-th wave, α_k is the random phase angle uniformly distributed from 0 to 2π, and $G(\phi_k)$ is the radiation gain of the antenna in the direction of the angle of incidence ϕ_k as shown in Fig. 1.

We note that C_k^2 represents a fraction of the signal power within $d\phi$ of the angle of incidence ϕ, for which we use a Laplacian distribution – a model observed in macrocellular sites [3]

$$C_k^2 = L(\phi) d\phi = Q \exp\left(-\sqrt{2} \frac{|\phi|}{\sigma} \right) d\phi \qquad (3)$$

where Q is a normalization factor chosen such that the integral of the distribution function is 1 over the range [−π, π], and σ is a close approximation of the angular spread of the signal.

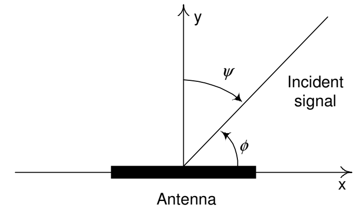


Fig. 1. Coordinate system for antenna and incident signal.

The cross covariance $\left\langle E_1\, E_2^* \right\rangle$ is then given by

$$\left\langle E_1\, E_2^* \right\rangle = E_0^2 \left\langle \sum_{k=1}^{K}\sum_{i=1}^{K} C_k C_i G_1(\phi_k) G_2^*(\phi_i) e^{j(\alpha_k-\alpha_i)} \right\rangle, \quad (4)$$

which can be simplified to the following equation since ϕ and α are statistically independent

$$\left\langle E_1\, E_2^* \right\rangle = E_0^2 \sum_{k=1}^{K}\sum_{i=1}^{K} \left\langle C_k C_i G_1(\phi_k) G_2^*(\phi_i) \right\rangle \left\langle e^{j(\alpha_k-\alpha_i)} \right\rangle. \quad (5)$$

The term $\left\langle e^{j(\alpha_k-\alpha_i)} \right\rangle$ equals zero, unless $k = i$, then

$$\left\langle E_1 E_2^* \right\rangle = K E_0^2 \left[\left\langle C_k^2 G_1(\phi) G_2^*(\phi) \right\rangle \right], \quad (6)$$

from which we obtain the following terms

$$\left\langle E_1 E_2^* \right\rangle = K E_0^2 \int_{-\pi}^{\pi} L(\phi) G_1(\phi) G_2^*(\phi) d\phi \quad (7)$$

$$\left\langle E_1\, E_1^* \right\rangle = K E_0^2 \int_{-\pi}^{\pi} L(\phi) \left| G_1(\phi) \right|^2 d\phi. \quad (8)$$

Equation (8) is the signal power received by a beam. To arrive at the normalized correlation coefficient, we substitute (7) and (8) into (1), and note that $\left\langle E_1 \right\rangle = \left\langle E_2 \right\rangle = 0$, hence,

$$\rho = \frac{\int_{-\pi}^{\pi} L(\phi) G_1(\phi) G_2^*(\phi) d\phi}{\sqrt{\int_{-\pi}^{\pi} L(\phi) \left| G_1(\phi) \right|^2 d\phi} \sqrt{\int_{-\pi}^{\pi} L(\phi) \left| G_2(\phi) \right|^2 d\phi}}. \quad (9)$$

III. Correlation Coefficient and Average power in a Four-Beam Antenna System

A. Far-Field Patterns for a Four-beam Antenna System

The performance of angular diversity is evaluated in a four-beam antenna system chosen for its practicality and simplicity. This system can be designed using an eight-element uniform linear array (N = 8) where each element is fed with an equal current of progressive phase shift obtained from a Butler matrix. Four of the eight ports of the Bultre matrix are used to obtain the innermost beams, which have a progressive phase shift δ between the array elements of 3π/8, π/8, −π/8, −3π/8.

The array factor for an N-element uniform linear array backed by a ground plane placed λ/2 away, is given by [4]

$$G(\phi) = e^{j\frac{(N-1)}{2}(\delta + kd\cos\phi)} \frac{\sin\left[\left(\frac{N}{2}\right)(\delta + kd\cos\phi)\right]}{\sin\left(\frac{\delta + kd\cos\phi}{2}\right)} \left(1 - e^{jk\frac{\lambda}{2}\sin\phi}\right) \quad (10)$$

where $k = 2\pi/\lambda$ is the wave number, λ is the carrier wavelength, and d is the distance between two elements. Selecting d = 2λ/5 results in an antenna system where beams are centered at ϕ = 62°, 81°, 99°, and 118°, as shown in Fig. 2. Each of the two inner and outer beams has a half-power beamwidth of about 16° and 18°, respectively, for an aggregate of about 68°: a common value for sectored antennas deployed in urban wireless networks.

B. Correlation Coefficient and Received Power for a Four-beam Antenna System

The array factor given in (10) is used in (9) to determine the correlation coefficients between beams, and in (8) to determine the power received by each beam. The range of the integrals reduces to [0,π] as the field behind the ground plane is zero.

Figs. 3 and 4 show the composite correlation coefficient and power ratio between the best and second-best beams for incident signals of varying angular spread, respectively. The best and second-best beams were selected on the basis of signal power, and were continuously updated as the angle of incidence changed. For each angular spread, the signal angle of incidence ψ varied between −60° to 60°, the equivalent of a base station sector.

We note that the correlation coefficient is lowest when the signal angle of incidence is at the center of a beam, as expected due to the orthogonal property of beams generated by the Butler matrix. Alternatively, the power ratio is lowest when the signal

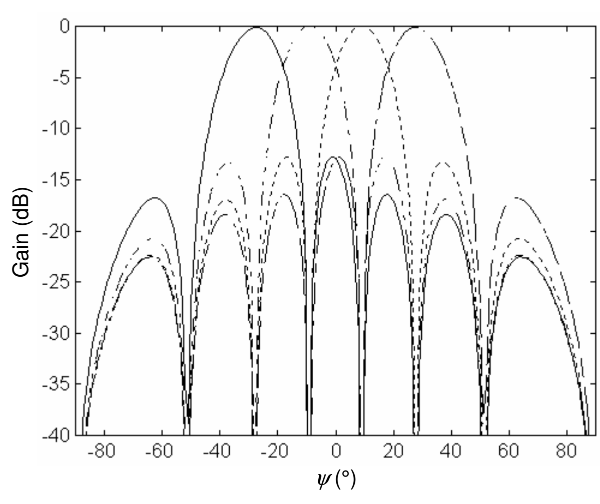


Fig. 2. Normalized gain for a four-beam antenna system.

angle of incidence is at the point of intersection between two beams – exactly the angle of maximum correlation between the two beams. Furthermore, the wider the angular spread of the incident signal, the lower the variations in correlation coefficient and power ratio between beams. The aforementioned factors lend a notion to variations in the performance of angular diversity depending on the angle of incidence and angular spread of the incident signal: factors determined by the location of the mobile unit and the propagation environment.

IV. Diversity Combining Performance

Maximal-ratio combining is used as an example of the expected performance of angular diversity. First, the general case of four-branch MRC is presented, followed by the case of two-branch MRC, as this number is common in commercial base stations. In this section, a 'beam' constitutes a 'branch' and two terms are used interchangeably. Furthermore, the spatial distribution of noise is taken to be uniform; therefore, noise power is equal across the four beams.

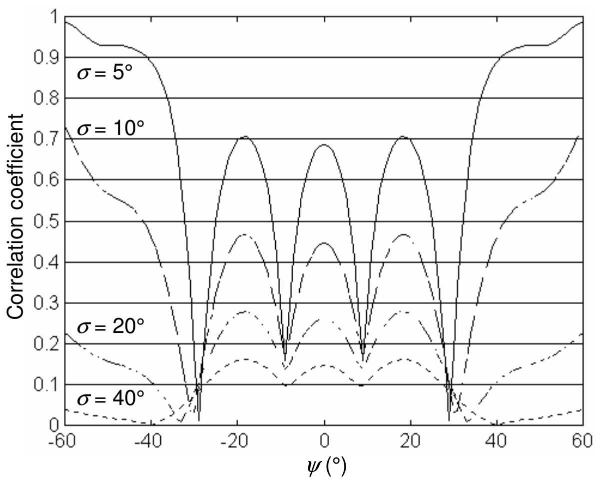


Fig. 3. Composite correlation coefficient between best and second-best beams of a four-beam antenna system for incident signals of varying angular spread.

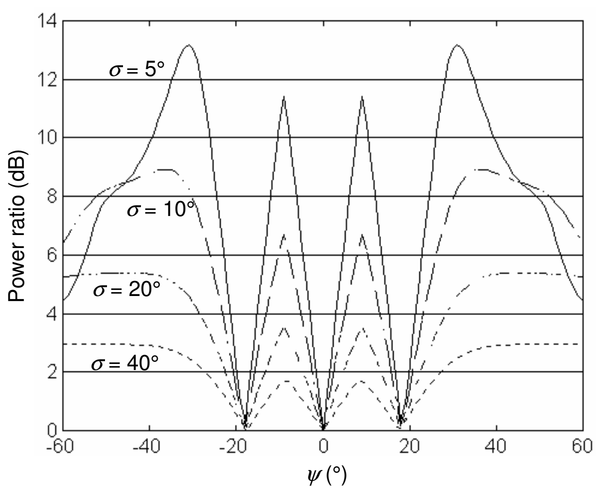


Fig. 4. Composite power ratio between best and second-best beams of a four-beam antenna system for incident signals of varying angular spread.

A. Four-Branch Maximal-Ratio Combining

The probability density function (pdf) and cumulative distribution function (cdf) of the instantaneous SNR γ for an M-branch MRC are given by [5]

$$p(\gamma) = \frac{1}{\prod_{m=1}^{M}\lambda_m}\sum_{m=1}^{M}\frac{\exp\left(-\gamma/\lambda_m\right)}{\prod_{k\neq m}^{M}\left(1/\lambda_m - 1/\lambda_k\right)} \tag{11}$$

$$P(\gamma \le x) = 1-\sum_{m=1}^{M}\frac{(\lambda_m)^{M-1}\exp\left(-x/\lambda_m\right)}{\prod_{k\neq m}^{M}\left(\lambda_m-\lambda_k\right)} \tag{12}$$

where λ_m are the M eigenvalues of the $M \times M$ covariance matrix whose entries are given by (7) divided by the branch noise power.

Fig. 5 shows the cdf of a four-branch MRC combining as a function of γ / Γ, where Γ is the average SNR of the beam with strongest power. An incident signal of 5° and 20° angular spread impinges at 0° (i.e. broadside, where beams 2 and 3 are at equal gain) and at 9° (the center of beam 3). The performance curves of the limit case of very narrow (e.g. $\sigma = 0.15°$) and very wide (e.g. uniformly distributed) incident signal are illustrated for comparison. In the case of narrow angular spread, the performance degrades to that of a two, correlated-branch MRC when the signal angle of incidence is 0°, while the performance for an angle of incidence of 9° it is that of a single branch. In the case of wide angular spread, we obtain the performance of MRC with four independent branches.

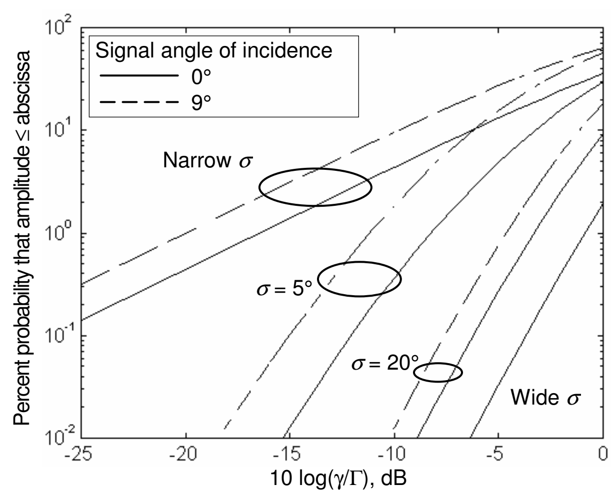


Fig. 5. Probability distribution of instantaneous SNR γ for a four-beam antenna system with maximal-ratio combining. Γ is the average SNR of the strongest beam.

B. Two-Branch Maximal-Ratio Combining

In the case of a two-branch MRC, (12) reduces to the following simplified form

$$P(\gamma \le x) = \frac{c_1}{c_1 - c_2}\left(1 - e^{-x/\Gamma c_1}\right) + \frac{c_2}{c_2 - c_1}\left(1 - e^{-x/\Gamma c_2}\right) \quad (13)$$

where,

$$c_1 = \frac{1}{2}\left(1 + \frac{1}{m} - \sqrt{\left(1 + \frac{1}{m}\right)^2 - \frac{4(1-\rho)}{m}}\right), \quad (14)$$

$$c_2 = \frac{1}{2}\left(1 + \frac{1}{m} + \sqrt{\left(1 + \frac{1}{m}\right)^2 - \frac{4(1-\rho)}{m}}\right), \quad (15)$$

m is the average SNR difference between the two branches, and ρ is the correlation coefficient between two beam signals (envelope correlation).

Fig. 6 shows the performance of maximal-ratio combining for branch SNR difference levels between 0 dB and 24 dB for uncorrelated signals (ρ = 0), signals with a correlation coefficient of 0.5, and perfectly correlated signals (ρ = 1). In the cases of ρ = 0 and 1, we evaluate (13) in the limit as ρ approaches 0 and 1 to avoid singularities.

C. Bit Error Rate Performance for a BPSK Signal

The probability of error in a slow, flat fading channel is

$$P_e = \int_0^\infty P_e(\gamma)\, p(\gamma)\, d\gamma \quad (16)$$

where $p(\gamma)$ is the pdf of the instantaneous SNR γ due to the fading channel and is obtained by taking the derivative of (13), and $P_e = Q\left(\sqrt{2\gamma}\right)$ is the probability of error for BPSK.

Fig. 7 shows the bit-error-rate performance for a BPSK signal with maximal-ratio combining for branch SNR difference levels between 0 dB and 24 dB for uncorrelated signals (ρ = 0), and perfectly correlated signals (ρ = 1).

V. General Discussion

Diversity reception is commonly used to mitigate the effects of fast fading on a desired signal, and thereby to increase the average SNR of the received signal by mainly increasing the average carrier power. Smart antennas play a complementary role: they seek to increase the average SNR of a desired signal by reducing interference. In combining diversity reception and smart antennas into a single system, it is desired to achieve the maximum possible SNR: a critical parameter that ultimately determines network capacity and quality (Shannon's equation).

The performance of diversity reception is a function of the average SNR in each branch of the diversity combiner and the correlation coefficient between their respective signals. In a mobile environment these two variables are the result of several factors that include the spatial distribution of interference and received signal strength. In addition, when angular diversity is

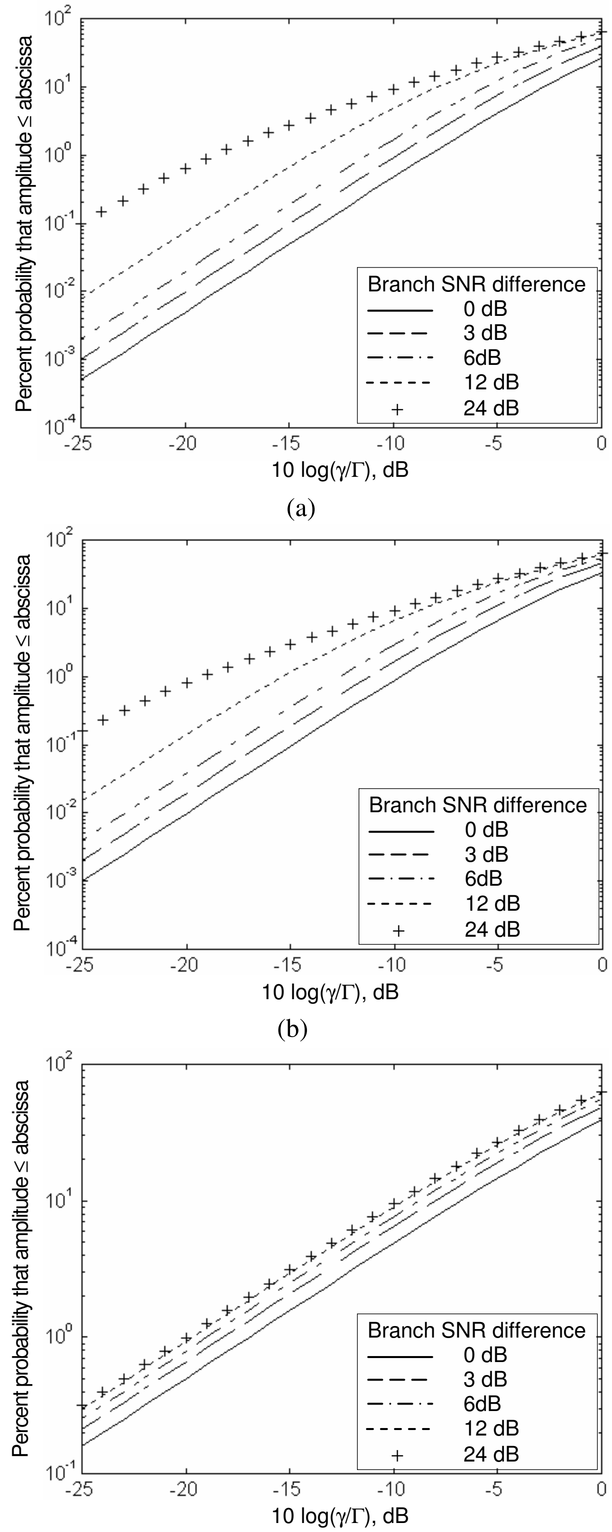
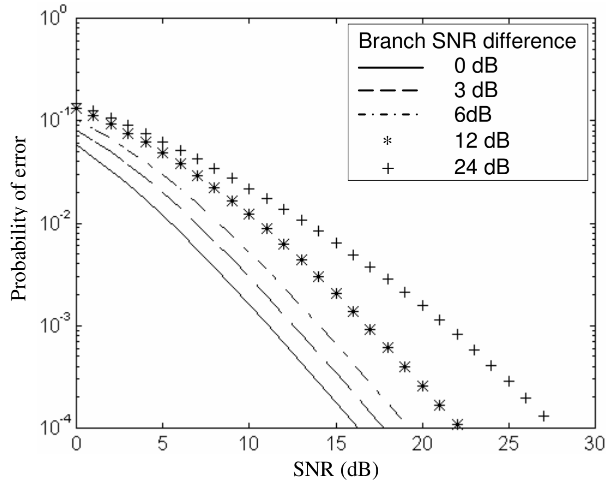
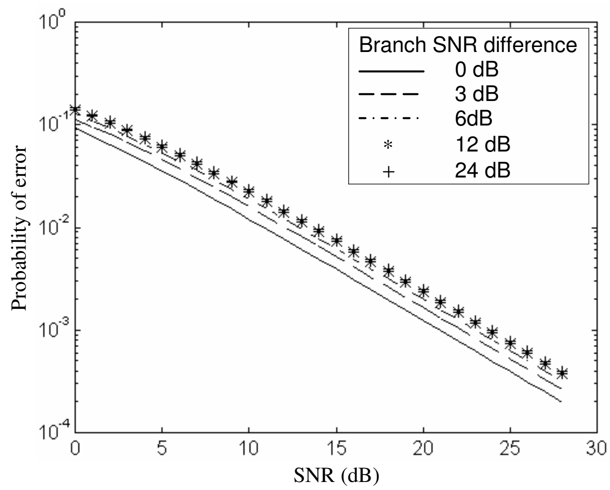


Fig. 6. Probability distribution of instantaneous SNR γ for a two, unequal branch MRC (a) uncorrelated branches, (b) ρ = 0.5, and (c) perfectly correlated branches. Γ is the average SNR of the stronger branch.



(a)



(b)

Fig. 7. Bit error rate performance for BPSK signal in slow, flat fading Rayleigh channel with two unequal-branch MRC (a) non-correlated branches and (b) perfectly correlated branches.

deployed, the angular spread and the angle of incidence of the desired and interfering signals and the half-power beamwidth of each beam of a multibeam antenna become important contributing factors.

The angular spread of the received signal is largely a function of the height of the base station antenna, and of the morphology and clutter surrounding it. The angular spread and the angle of arrival would determine the power received by each beam of the multibeam antenna system. Because of these variables, angular diversity in a land mobile environment provides an interesting trade-off between relative branch power and correlation coefficient. At the center of a beam, angular diversity provides maximum decorrelation among branch signals, but also the largest power difference due to the orthogonal property of the beams. Meanwhile, the reverse is true at the points where two beams intersect: correlation coefficient is at its highest and the power difference between branches is zero.

Signals characterized by a wide angular spread are generally encountered in base station antennas located below the surrounding clutter. Results show that in this case the correlation coefficient is more uniform across the narrow beams, and the power ratio between adjacent beams is reduced. This does not necessarily imply an enhancement in the performance of angular diversity since a narrower beam would collect a smaller fraction of the power of the incident signal.

Alternatively, signals characterized by a narrow angular spread are generally encountered in relatively high base stations, especially those in open areas. In this case, large differences in average SNR between the diversity branches as well as in correlation coefficient would limit the benefits of diversity combining.

Finally we note that angular diversity is generally at it weakest performance on the outer beams of a multibeam antenna system.

VI. Conclusions

A mathematical model for angular diversity is presented and applied to a four-beam antenna system. Results show that the correlation coefficient and branch power, two parameters that impact the performance of diversity reception, vary significantly as a function of the angular spread of the incident signal as well as the angle of incidence. The performance of the four-beam antenna system with a four-branch and a two-branch MRC under unequal branch SNR condition is presented to investigate the effects of such variations, along with the performance of a BPSK signal in a slow, flat Rayleigh fading channel.

References

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